**Human-Oriented Robotics**
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University of Freiburg
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**Submission:** Send your solution via email to `palmieri@informatik.uni-freiburg.de` until November 18, 2014 with subject "[`exercises`] `Sheet 3`". All files (Matlab scripts, exported figures, hand-written notes in pdf/jpg format) should be compressed into a single zip file named `lastname_sheet3.zip`.

# Exercise 3: Basics of Probabilistic Reasoning

## Exercise 3.1: Probabilistic Graphical Models

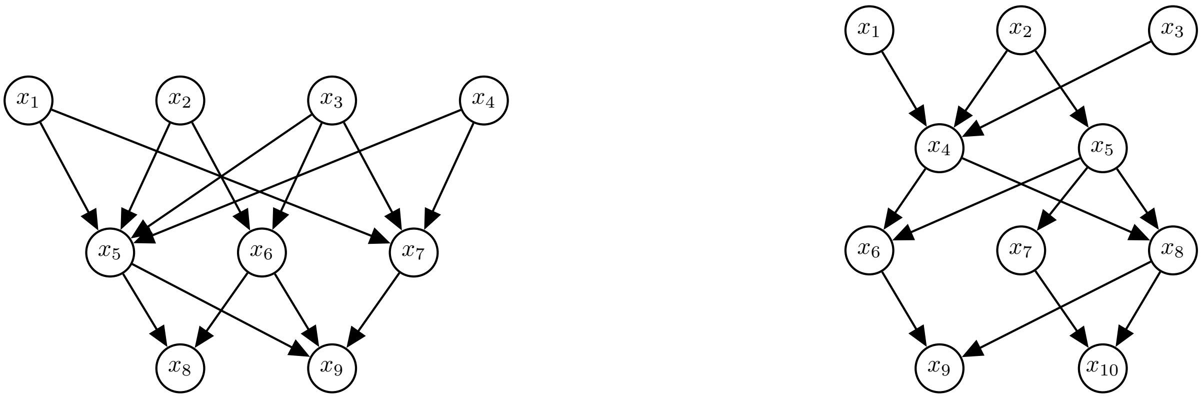


Figure 1: Probabilistic graphical models **a**(left) and **b**(right)

a) Consider a malfunctioning coffee-serving robot that you have to repair. From your experience you can state the following: a loose cable is a possible cause for loss of commands over the bus that controls the robot's arm and is also an explanation for high CPU load of the robot's built-in embedded computer (because, for example, a certain task throws an exception and restarts constantly). In turn, either of these could cause the arm to malfunction and spill coffee. An increased temperature of the robot's PC can also be explained by a high CPU load.
Represent these causal links in a probabilistic graphical model. Let $a$ stand for Loose-Cable, $b$ for Loss-of-Commands, $c$ for High-CPU-Load, $d$ for Spilled-Coffee, and $e$ for Increased-PC-Temperature.

b) From the given directed graphs **a** and **b** shown in Figure 1, find the corresponding distribution. Remember that:

$$p(\mathbf{x}) = \prod_{k=1}^{K} p(x_k|\mathrm{pa}_k) \tag{1}$$

## Exercise 3.2: Markov Chains

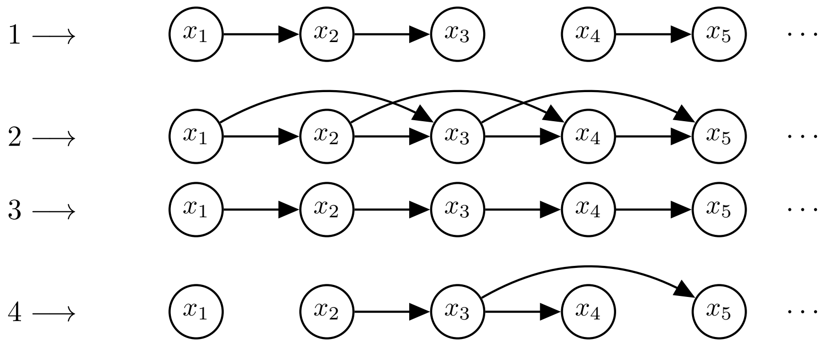


Figure 2: Markov Chains

a) Find the order of all Markov Chains shown in Figure 2

**Exercise 3.3: State-space models** Consider the model in Fig. 3 which represents a Markov decision process showing evolution of states, actions and rewards; derive the full joint distribution of the variables of the model i.e. $p(s_1, \ldots, s_k, a_1, \ldots, a_k, r_1, \ldots, r_k)$ assuming the following distributions are given $p(s_i|s_{i-1}, a_{i-1})$, $p(a_i|s_i)$ and $p(r_i|s_i, a_i)$.

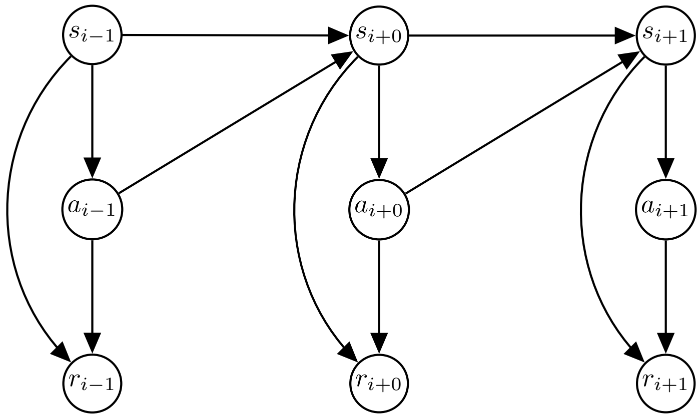


Figure 3: Markov Decision Process model

**Exercise 3.4: Joint Distribution** Assume $\mathbf{x} = \{x_1, x_2, ..., x_K\}$ are discrete random variables. By marginalizing out the variables in order, show that the representation

$$p(\mathbf{x}) = \prod_{k=1}^{K} p(x_k|\mathrm{pa}_k) \tag{2}$$

for the joint distribution of a direct graph is correctly normalized, i.e.

$$\sum_{x_1} \sum_{x_2} \cdots \sum_{x_K} p(\mathbf{x}) = 1,$$

provided that all conditional distributions $p(x_k|\mathrm{pa}_k)$ are correctly normalized. For example, for the $K$-th node

$$\sum_{x_K} p(x_K|\mathrm{pa}_K) = 1$$