



Human-Oriented Robotics
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 University of Freiburg
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Submission: Send your solution via email to palmieri@informatik.uni-freiburg.de until November 11, 2014 with subject “[exercises] Sheet 2”. All files (Matlab scripts, exported figures, hand-written notes in pdf/jpg format) should be compressed into a single zip file named `lastname_sheet2.zip`.

Exercise 2: Introduction to Probability

Exercise 2.1: Conditional Probability

- We have three colored boxes r (red), b (blue), and g (green). Box r contains 3 apples, 4 oranges, and 3 limes, box b contains 1 apple, 1 orange and 0 limes, and box g contains 3 apples, 3 oranges and 4 limes. If a box is chosen at random with probabilities $p(r) = 0.2$, $p(b) = 0.2$, $p(g) = 0.6$, and a piece of fruit is removed from the box (with equal probability of selecting any of the items in the box), then what is the probability of selecting an apple?
- If we observe that the selected fruit is in fact an orange, what is the probability that it came from the green box?

Exercise 2.2: Bayes’ Rule

- Suppose we live in a world with only three possible robot positions $X = (x_1, x_2, x_3)$. Let Z be a Boolean sensor variable characterized by the following probabilities:

$$\begin{aligned} p(z|x_1) &= 0.8 & p(\neg z|x_1) &= 0.2 \\ p(z|x_2) &= 0.4 & p(\neg z|x_2) &= 0.6 \\ p(z|x_3) &= 0.1 & p(\neg z|x_3) &= 0.9 \end{aligned}$$

Suppose that the marginal distribution of the robot position is uniform, $p(x_i) = \frac{1}{3}$. Calculate the posterior $p(x_i|z)$ for each of the locations $X = (x_1, x_2, x_3)$.

Exercise 2.3: Expectation, Variance and Covariance

- The two variables x and z are statistically independent. Show that the mean and variance of their sum satisfies

$$E[x + z] = E[x] + E[z] \tag{1}$$

$$\text{Var}[x + z] = \text{Var}[x] + \text{Var}[z] \tag{2}$$

- b) Show that if two variables x and y are independent, then their *covariance* is zero. The covariance is defined as

$$\text{Cov}[x, y] = \text{E}[(x - \mu_x)(y - \mu_y)] \quad (3)$$

- c) You have written a program which produces a *covariance matrix* as follows. Is this a valid covariance matrix?

$$C = \begin{bmatrix} 9 & 4 \\ 4 & 1 \end{bmatrix} \quad (4)$$

Exercise 2.4: Probability distributions and moments

This exercise requires you to import the provided Matlab file `distributions.mat` into your Matlab workspace. It will then contain a 10000×5 matrix called `data`.

- a) Each of the five columns of *data* contains 10.000 values that were randomly sampled from unknown probability distributions. *Without* using the built-in functions, calculate the
- mean
 - variance
 - standard deviation
 - 3rd central moment
 - 4th central moment

for each column. Use vectorization, no for-loops.

- b) Now verify your results using the built-in functions `mean`, `var`, `std` and `moment`.
- c) Plot each of the five distributions into the same figure using the `plot` command. Use $[-5, 20]$ as the x-axis limits and `'-'` as the line style. You can use the `histc` command with e.g. 100 bins to compute a histogram of the relative frequencies of the values in each column of `data`. Normalize the frequencies such that for each distribution, they sum up to 1.0.
- d) By looking at the plot, which data set do you think was sampled from which type of probability distribution explained in the lecture?
- e) If you suspect any of the distributions to be Gaussian, what were the parameters originally used to build the distribution?