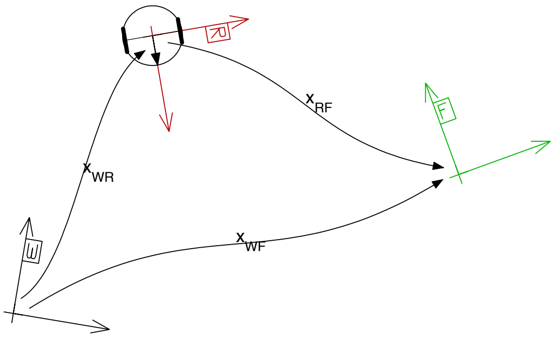
librobotics Reference



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Contents

1 About librobotics

For the sake of brevity, here just the facts:

- librobotics is a small library with frequently used Octave/Matlab functions in Robotics, especially for visualization.
- All commands are fully documented, just type `help command.`
- librobotics is compatible with both, Matlab and Octave.
- It's open source, feel free to distribute and extend.
- librobotics was written by Kai Arras mainly in 2003-4 as part of the CAS Robot Navigation Toolbox. Minor adaptations since then.
- librobotics can be downloaded from the Social Robotics Lab homepage at `http://srl.informatik.uni-freiburg.de/downloads`

Enjoy!

2 Reference

2.1 `chi2invtable`

Lookup table of the inverse of the χ_2 cumulative distribution function.

`x = chi2invtable(p,v)` returns the inverse of the χ^2 cumulative distribution function (cdf) with `v` degrees of freedom at the value `p`. The χ^2 cdf with `v` degrees of freedom, is the Γ cdf with parameters `v/2` and `2`.

Opposed to `chi2inv` of the Matlab statistics toolbox (which might be not part of your Matlab installation), `chi2invtable` is a lookup table and thereby much faster than `chi2inv`. However, as any lookup table is a collection of sample points, accuracy is smaller. Between the sample points of the cdf, a linear interpolation is made.

Currently, the function supports the degrees of freedom `v` between 1 and 10 and the probability levels `p` between 0 and 0.9999 in steps of 0.0001 plus the level 0.99999.

Example: A typical usage scenario of `chi2invtable` (or `chi2inv`) is during the matching step of a Kalman filter localization or slam cycle. Given the probability α and features of dimension n, `chi2invtable` yields the maximal Mahalanobis distance (or gate distance) $\chi^2_{n,\alpha}$ which a candidate pairing with innovation ν_{ij} and innovation covariance S_{ij} may have in order to be accepted. In other words, the pairing is accepted if the following holds:

$$\nu_{ij}^T \, S_{ij} \, \nu_{ij} < \chi^2_{n,\alpha}$$

See also `chi2inv`.

2.2 compound

Compound relationship in 2D.

`xik = compound(xij,xjk)` returns the compound relationship of the two 2D transforms `xij` and `xjk` which are arranged head-to-tail. `x`'s are 3×1-vectors $[x, y, \theta]^T$, orientations within $[0..2\pi[$.

Example: Given the transform x_{WR} which expresses entity R in the reference frame of W and transform x_{RS} which represents entity S in the frame of R, then the composition x_{WS} is the relationship which expresses S in the frame of W:

$$x_{WS} = x_{WR} \oplus x_{RS}$$

Note that the compound operation would be the same as vector addition if there were no orientations.

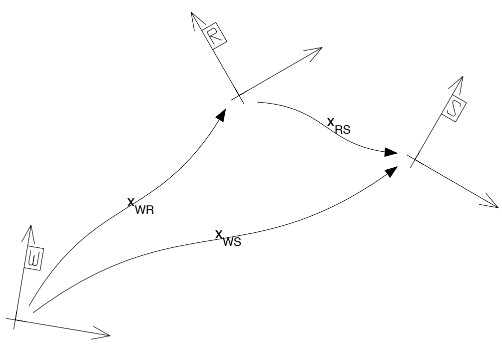

Figure 1: `compound.m`

Reference: R. Smith, M. Self, P. Cheeseman, "Estimating Uncertain Spatial Relationships in Robotics," in Autonomous Robot Vehicles, I.J. Cox and G.T. Wilfong, Eds.: Springer-Verlag, 1990, pp. 167-193.

See also `icompound`, `j1comp`, `j2comp`.

2.3 `diffangle`

Take difference of two angles and unwrap it.

α = `diffangle(`α_1`,`α_2`)` determines the minimal difference $\alpha = \alpha_1 - \alpha_2$ between two angles α_1 and α_2. If either α_1 or α_2 is `Inf`, `Inf` is returned.

Example: The difference of $\alpha_1 = 107.54$ and $\alpha_2 = -115.97$ is $\alpha = -136.49$.

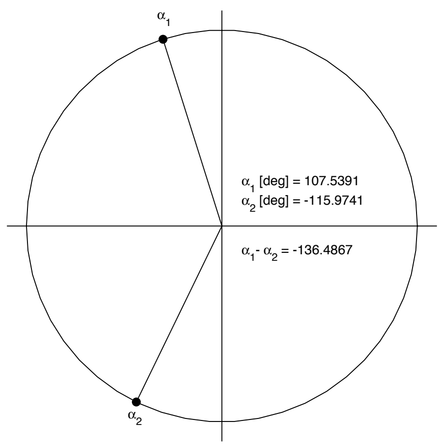

Figure 2: `diffangle.m`

See also `normangle`.

2.4 `normangle`

Put angle into a 2π interval.

`ar = normangle(a,min)` puts angle `a` into the interval [`min`..`min`+2π[. If `a` is `Inf`, `Inf` is returned.

See also `diffangle`.

2.5 drawarrow

Draw an arrow.

drawarrow(xs,xe,filled,hsize,color) draws an arrow from xs to xe. The first two elements of xs, xe are interpreted as the x- and y-positions. filled enables and disables head filling, hsize scales the size of the head in [m], and color is a [r g b]-vector or a color string such as 'r' or 'g'.

h = drawarrow(...) return a column vector of handles to the graphic objects of the arrow drawing.

Example: The commands

```
drawarrow([1 3 -pi/1.8],[2 0 pi/30],0,1,'k');
drawarrow([-3 2 -pi/8],[-2 1 3*pi/2],0,0.3,'b');
drawarrow([0.5 -0.5],[0 2.2],1,1,[0.4 0.9 0.1]);
drawarrow([-1 2], [-2 -1],1,0.2,[0.6 0.6 0.2]);
h = drawarrow([0 -1],[-1 -1.6],1,0.5,'r');
set(h,'LineWidth',3);
```

generate the arrows shown in the figure below. Note that the line width of the last arrow in the bottom of the figure has been changed using the handle vector h.

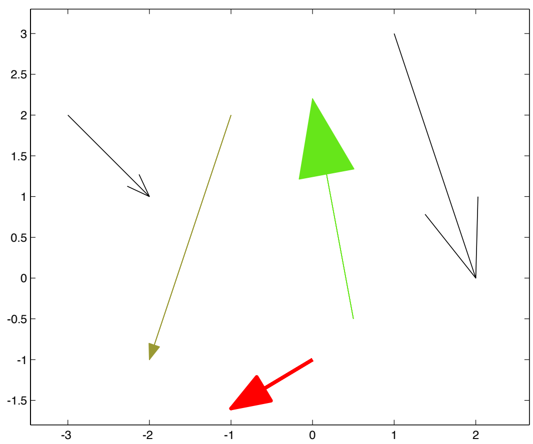

Figure 3: drawarrow.m

See also drawreference, plot.

2.6 `drawellipse`

Draw ellipse.

`drawellipse(x,a,b,color)` draws an ellipse at `x` = $[x, y, \theta]^T$ with half axes `a` and `b`. Orientation θ is the inclination angle of `a`, regardless if `a` is smaller or greater than `b`. `color` is a [r g b]-vector or a color string such as `'r'` or `'g'`.

`h = drawellipse(...)` returns the graphic handle `h`.

Example: The commands

```
plot(1,-0.4,'k+');
drawellipse([1 -0.4 pi/6],1,0.5,'k')
plot(0.5,0,'k+');
drawellipse([0.5 0 pi/6],0.25,0.5,'b');
plot(1.8,-0.8,'k+');
h = drawellipse([1.8 -0.8 pi/6],0.2,0.2,[0 0.7 0.3]);
set(h,'LineStyle','-.');
```

generate the ellipses shown below. Note that the line style of the circle can be changed using the handle vector `h`.

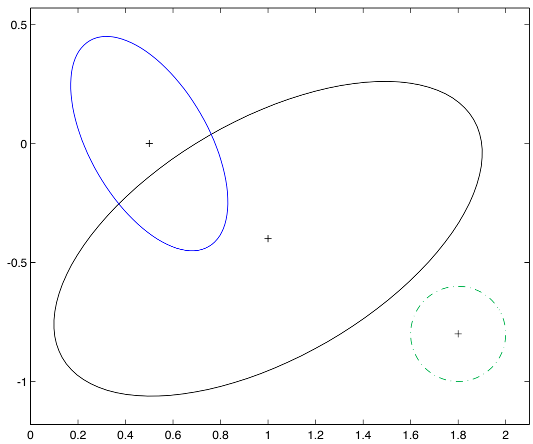

Figure 4: `drawellipse.m`

See also `drawprobellipse`.

2.7 `drawlabel`

Draw scalable text.

`drawlabel(x,str,scale,offset,color)` draws scalable text `str` at pose `x =` $[x, y, \theta]^T$ imitating the OCR font. With `scale` $= 1$, the height of the letters is 1 meter. `offset` shifts the text in [m] from the x, y position in postive x- and y-direction. `color` is either a [r g b]-vector or a color string such as `'r'` or `'g'`.

Currently, the following characters are implemented: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, W, R, S, E, F, M, P.

`h = drawlabel(...)` returns a column vector of handles to all line objects of the drawing, one handle per line.

Example: The commands

```
plot(-3,2,'k+'); drawlabel([-3 2 0],'123',0.5,0.0,'k');
plot(-3,1,'k+'); drawlabel([-3 1 0],'123',0.5,0.1,'k');
plot(-3,0,'k+'); drawlabel([-3 0 0],'123',0.5,0.2,'k');
plot(-1,2,'k+'); drawlabel([-1,2 -2.8],'R1',0.2,0.1,[.5 .5 .5]);
plot( 1,3,'k+'); drawlabel([ 1 3 -pi/1.8],'F30493',0.8,0.3,'g');

plot(-2,-1,'k+'); h = drawlabel([-2 -1 2.6],'S04',0.3,0.1,'r');
set(h,'LineWidth',3);
plot(-1,-2,'k+'); h = drawlabel([-1 -2 pi/3],'REF',0.3,0.1,'m');
set(h,'LineWidth',2);
```

generate the figure shown below. Note that line width can be varied using the handle vector `h`.

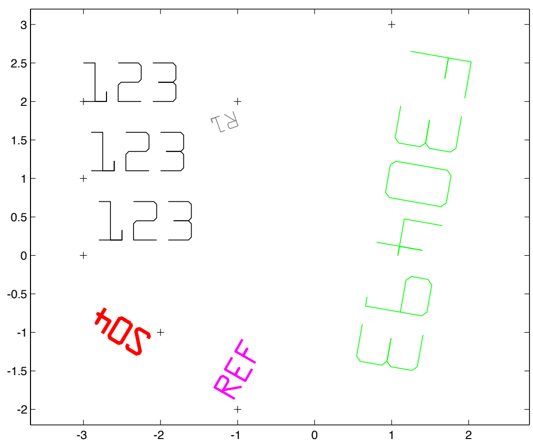

Figure 5: `drawlabel.m`

See also `text`.

2.8 drawprobellipse

Draw elliptic probability region of a Gaussian in 2D.

`drawprobellipse(x,C,alpha,color)` draws the elliptic iso-probability contour of a Gaussian distributed bivariate random vector `x` at the significance level `alpha`. The ellipse is centered at `x` = $[x, y]^T$ where `C` is the associated 2×2 covariance matrix. `color` is a [r g b]-vector or a color string such as `'r'` or `'g'`.

`x` and `C` can also be of size 3×1 and 3×3 respectively.

The function uses `chi2invtable` instead of `chi2inv` from the Matlab statistics toolbox.

In case of a negative definite matrix `C`, the ellipse collapses to a line which is drawn instead.

`h = drawprobellipse(...)` returns the graphic handle `h`.

Example: The commands

```
x1 = [ 1,2]; C1 = [0.25 -0.2; -0.2 0.3];
drawprobellipse(x1,C1,0.95,'k');
x2 = [-1,0]; C2 = [0.3 -0.03; -0.03 0.01];
drawprobellipse(x2,C2,0.95,'k');
x3 = [-2,3]; C3 = [0.01 0.005; 0.005 0.015];
drawprobellipse(x3,C3,0.95,'k');
```

generate the ellipses shown below. Note that line width can be varied using the handle vector `h`.

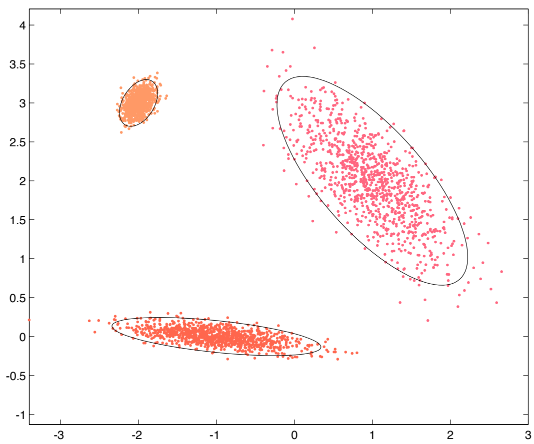

Figure 6: `drawprobellipse.m`

See also `drawellipse`, `chi2invtable`, `chi2inv`.

2.9 `drawreference`

Draw coordinate reference frame.

`drawreference(x,label,size,color)` draws a reference frame at pose `x` = $[x, y, \theta]^T$ and labels it with the string `label`. `size` is the length of the frame axes in [m], and `color` is a [r g b]-vector or a color string such as `'r'` or `'g'`.

`h = drawreference(...)` returns a column vector of handles to all graphic objects of the drawing. Remember that not all graphic properties apply to all types of graphic objects. Use `findobj` to find and access the individual objects.

Example: The commands

```
drawreference([0 2 pi/8],'W',1,'k');
drawreference([0 0 pi/3],'R2',0.4,'b');
drawreference([0.8 1.3 -1.8],'S43',0.5,[.8 .5 .1]);
drawreference([2 2.3 -0.3],'',0.6,[.6 .6 .6]);

h = drawreference([2 0.7 pi/9],'98',0.8,[.3 .7 .3]);
set(h,'LineWidth',2);
```

generate the figure shown below. Note that line width, line style and color can be varied using the handle vector `h`.

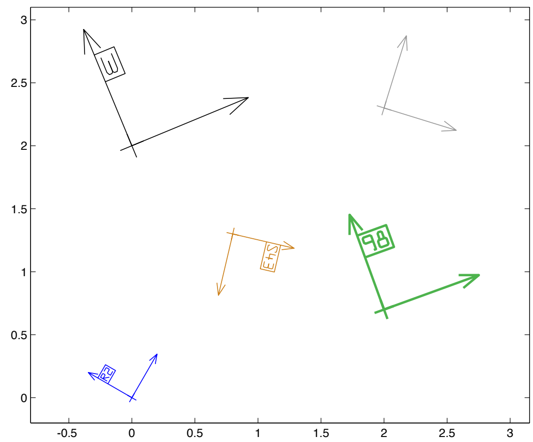

Figure 7: `drawreference.m`

See also `drawarrow`, `drawlabel`, `findobj`, `plot`.

2.10 `drawrobot`

Draw robot.

`drawrobot(x,color)` draws a robot at pose `x =` $[x, y, \theta]^T$ such that the robot reference frame is attached to the center of the wheelbase with the x-axis looking forward. `color` is a [r g b]-vector or a color string such as `'r'` or `'g'`.

`drawrobot(x,color,type)` draws a robot of type `type`. Five different models are implemented:

`type` $= 0$ draws only a cross with orientation θ
`type` $= 1$ is a differential drive robot without contour
`type` $= 2$ is a differential drive robot with round shape
`type` $= 3$ is a round shaped robot with a line at θ
`type` $= 4$ is a differential drive robot with rectangular shape
`type` $= 5$ is a rectangular shaped robot with a line at θ

`drawrobot(x,color,type,w,l)` draws a robot of type `type` with width `w` and length `l` in [m].

`h = drawrobot(...)` returns a column vector of handles to all graphic objects of the robot drawing. Remember that not all graphic properties apply to all types of graphic objects. Use `findobj` to find and access the individual objects.

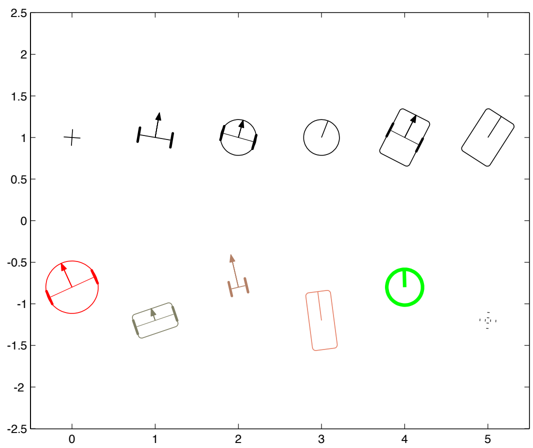

Figure 8: `drawrobot.m`

Example: The commands

```
drawrobot([0 1 1.5],'k',0);
drawrobot([1 1 1.4],'k',1);
drawrobot([2 1 1.3],'k',2);
drawrobot([3 1 1.2],'k',3);
drawrobot([4 1 1.1],'k',4);
drawrobot([5 1 1.0],'k',5);

drawrobot([0 -0.8 2.0],'r',2,0.6,0.6);
drawrobot([1 -1.2 1.9],[.5 .5 .4],4,0.5,0.3);
drawrobot([2 -0.8 1.8],[.7 .5 .4],1,0.2,0.8);
```

```
drawrobot([3 -1.2 1.7],[.9 .5 .4],5,0.3,0.7);

h = drawrobot([4 -0.8 1.6],'g',3,0.4,0.1);
set(h,'LineWidth',3);
h = drawrobot([5 -1.2 1.5],'k',0,0.4,0.1);
set(h,'LineWidth',2,'LineStyle',':');
```

generate the robots shown above. Note that line width, line style and color can be varied using the handle vector `h`.

See also `drawrect`, `drawarrow`, `findobj`, `plot`.

2.11 drawrect

Draw rounded rectangle.

`drawrect(x,w,h,r,filled,color)` draws a rectangle with round corners of radius `r`, width `w` and height `h`, centered at pose `x` where `x` is the 3×1 vector $[x, y, \theta]^T$. With `filled = 1` the rectangle is filled with color `color`, with `filled = 0` only the contour is drawn. `color` is a [r g b]-vector or a Matlab color string such as `'r'` or `'g'`.

Note that 2`r` must be greater or equal than the smaller of the two values `w`, `h`. For 2`r` = `w` = `h`, `drawrect` draws a circle.

`h = drawrect(...)` returns the graphic handle `h`.

Example: The commands

```
drawrect([0.4 1 2.6],1,0.6,0.2,0,'b');
drawrect([1.9 2 2.5],1,1.2,0.1,1,[.4 .9 .0]);
drawrect([1.9 2 2.5],1,1.2,0.1,0,[.2 .7 .0]);
drawrect([3.0 1 2.4],0.2,1.7,0.0,0,'r');
drawrect([4.0 0 2.3],0.4,0.4,0.2,1,[.8 .8 .8]);

h = drawrect([4 0 2.3],0.4,0.4,0.2,0,'k');
set(h,'LineWidth',2);
h = drawrect([5 1 0.6],0.3,0.7,0.15,0,[.9 .7 .0]);
set(h,'LineWidth',4);
```

generates the figure shown below. Note that line width, line style and color can be varied using the handle vector `h`.

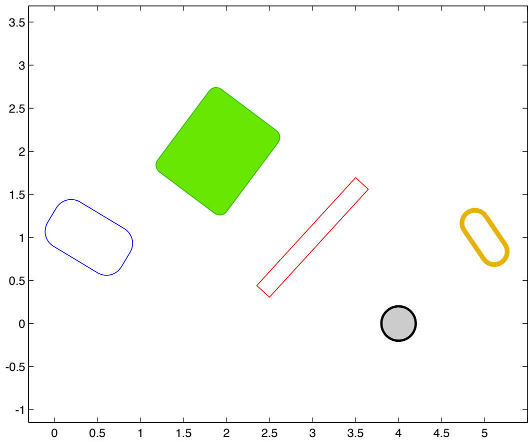

Figure 9: `drawrect.m`

See also `drawreference`, `plot`.

2.12 `drawtransform`

Illustrates a spatial relationship.

`drawtransform(xs,xe,shape,label,color)` draws a nice looking curved arrow from location `xs` (3×1) to `xe` (3×1) and labels it with the string `label`. `color` is a [r g b]-vector or a Matlab color string such as `'r'` or `'g'`. `shape` controls the shape of the curve: `'/'` for a S-shape, `'\'` for a Z-shape, `'('` for a left arc and `')'` for a right arc.

`h = drawtransform(...)` returns a column vector of handles to all graphic objects of the drawing. Remember that not all graphic properties apply to all types of graphic objects.

Example: The commands

```
plot(0,0,'k+'); plot(0.2, 1.5,'k+');
drawtransform([0 0],[0.2 1.5],'/','x1',[.9 .8 .0]);
plot(1,0,'k+'); plot(1.2, 1.5,'k+');
drawtransform([1 0],[1.2 1.5],'\','x2',[.7 .6 .0]);
plot(2,0,'k+'); plot(2.2, 1.5,'k+');
drawtransform([2 0],[2.2 1.5],'(','x3',[.5 .3 .0]);
plot(3,0,'k+'); plot(3.2, 1.5,'k+');
drawtransform([3 0],[3.2 1.5],')','x4',[.3 .0 .0]);

h = drawtransform([0 0],[2 0],')','','k');
set(h,'LineStyle','--');
h = drawtransform([1 0],[3 0],')','','k');
set(h,'LineStyle',':');
```

generate the arrows shown below. Note that line width, line style and color can be set using the handle vector `h`.

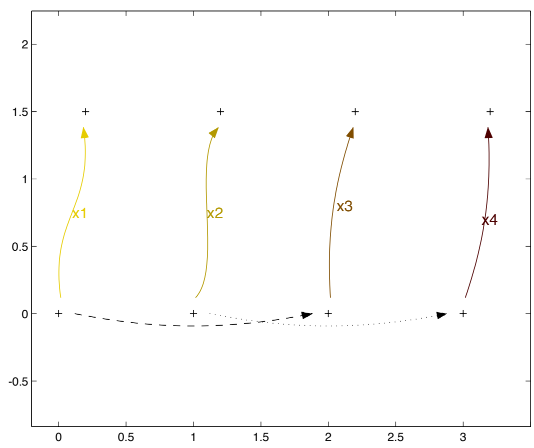

Figure 10: `drawtransform.m`

See also `drawreference`, `plot`.

2.13 `icompound`

Inverse 2D relationship.

`xji = icompound(xij)` returns the inverted 2D transform `xji` given the relationship `xij`. All `x`'s are 3×1-vectors, all angles within $[0..2\pi[$.

Example: Given a feature with attached frame F and the robot with attached frame R represented both in the world reference frame W by the transforms x_{WF} and x_{WR}, we look for F expressed in the robot reference frame R. The solution is the composition of the inverted x_{WR} with x_{WF}:

$$x_{RF} = \ominus x_{WR} \oplus x_{WF} \tag{1}$$

This frame transform is usually called *measurement prediction* in the context of feature-based Kalman filter localization or slam (with the simplifying assumption that sensor frame and robot frame coincide).

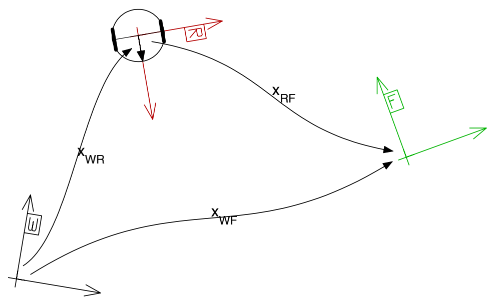

Figure 11: `icompound.m`

The figure has been generated by the following code:

```
xwr = [1.0, 2, -1.4];
xwf = [3.2, 1, pi/9];
drawreference([0 0 -pi/19],'W',0.7,'k');
drawreference(xwr,'R',0.7,[0.7 0 0]);
drawrobot(xwr,'k');
drawtransform(zeros(3,1),xwr,'\','x_W_R','k');
drawreference(xwf,'F',0.7,[0 0.7 0]);
drawtransform(zeros(3,1),xwf,'/','x_W_F','k');
drawtransform(xwr,xwf,'/','x_R_F','k');
```

Reference: R. Smith, M. Self, P. Cheeseman, "Estimating Uncertain Spatial Relationships in Robotics," in Autonomous Robot Vehicles, I.J. Cox and G.T. Wilfong, Eds.: Springer-Verlag, 1990, pp. 167-193.

See also `compound`, `jinv`.

2.14 `j1comp`

First Jacobian of the compound operator.

`J = j1comp(xi,xj)` returns the Jacobian matrix of the 2D composition of `xi` and `xj` derived with respect to the first operand `xi`. All `x`'s are 3×1-vectors, `J` is a 3×3-matrix.

The Jacobian is used to perform first-order error propagation when the input transforms $\hat{x}_{ij}$ and $\hat{x}_{jk}$ of the composition $\hat{x}_{ik} = \hat{x}_{ij} \oplus \hat{x}_{jk}$ are uncertain. To calculate the uncertainty of the output $\hat{x}_{ik}$, the compound operator is derivated with respect to the two operands yielding a 3×6 Jacobian matrix $J_{\oplus}$ which consists of a left 3×3 half, $J_{1\oplus}$, and a right 3×3 half, $J_{2\oplus}$.

$$J_{1\oplus} = \left.\frac{\delta \hat{x}_{ik}}{\delta x_{ij}}\right|_{\hat{x}_{ij}} \qquad J_{2\oplus} = \left.\frac{\delta \hat{x}_{ik}}{\delta x_{jk}}\right|_{\hat{x}_{jk}}$$

$$J_{\oplus} = [J_{1\oplus} \;\; J_{2\oplus}]$$

With C_{ijk} as the input covariance matrix

$$C_{ijk} = \begin{bmatrix} C_{ij} & C_{ijjk} \\ C_{jkij} & C_{jk} \end{bmatrix}$$

the covariance matrix of the output transform C_{ik} is given by the error propagation law

$$\begin{aligned} C_{ik} &= J_{\oplus} C_{ijk} J_{\oplus}^T \\ &= J_{1\oplus} C_{ij} J_{1\oplus}^T + J_{1\oplus} C_{ijjk} J_{2\oplus}^T + J_{2\oplus} C_{jkij} J_{1\oplus}^T + J_{2\oplus} C_{jk} J_{2\oplus}^T \end{aligned}$$

where the submatrix C_{ijjk} $(= C_{jkij}{}^T)$ is the cross-correlation between $\hat{x}_{ij}$ and $\hat{x}_{jk}$.

See also `j2comp`, `jinv`, `compound`, `icompound`.

2.15 `j2comp`

Second Jacobian of the compound operator.

`J = j2comp(xi,xj)` returns the Jacobian matrix of the 2D composition of `xi` and `xj` derived with respect to the second operand `xj`. All `x`'s are 3×1-vectors, `J` is a 3×3-matrix.

See explanation at `j1comp`.

See also `j1comp`, `jinv`, `compound`, `icompound`.

2.16 `jinv`

Jacobian of the inverse compound operator.

`J = jinv(xij)` returns the Jacobian matrix of the inversion `xji` of `xij`. All `x`'s are 3×1-vectors.

The Jacobian is used to perform first-order error propagation when the input transform $\hat{x}_{ij}$ of the inversion $\hat{x}_{ji} = \ominus \hat{x}_{ij}$ is uncertain. To calculate the uncertainty of the output $\hat{x}_{ji}$, the inverse compound operator is derivated with respect to the operand yielding the 3×3 Jacobian matrix $J_{\ominus}$.

$$J_{\ominus} = \left. \frac{\delta \hat{x}_{ji}}{\delta x_{ij}} \right|_{\hat{x}_{ij}}$$

With C_{ij} as the input covariance matrix, the covariance matrix of the output transform, C_{ji}, is given by the error propagation law

$$C_{ji} \quad = \quad J_{\ominus} C_{ij} J_{\ominus}^{T}$$

See also `j1comp`, `j2comp`, `compound`, `icompound`.

2.17 `mahalanobis`

Calculate the Mahalanobis distance.

`d = mahalanobis(v,S)` calculates the chi square distributed Mahalanobis distance given the innovation vector `v` and the innovation covariance matrix `S`.

Formally, with the innovation ν and the innovation covariance matrix S, the Mahalanobis distance is

$$D = \nu^T S \nu \tag{2}$$

The Mahalanobis distance is a quadratic form and allows to test on positive definiteness of a matrix S. With any $\nu \neq 0$, S is positive definite if $D > 0$ and positive semidefinite if $D \geq 0$.

See also `chi2invtable`, `chi2inv`.

2.18 meanwm

Multivariate weighted mean (Information Filter).

`[xw,Cw] = meanwm(x,C)` calculates the multivariate weighted mean. `x` is a matrix of dimension $m \times n$ where each column is interpreted as a Gaussian distributed random vector of dimension $m \times 1$. `C` is a $m \times m \times n$ matrix where each $m \times m$ matrix is interpreted as the covariance estimate associated to its respective row vector. The function returns the weighted mean vector `xw` and the weighted covariance matrix `Cw` of dimensions $m \times 1$ and $m \times m$ respectively.

The multivariate weighted mean is also known as the Information Filter (IF), a batch formulation of the (recursive) Kalman filter. Given the random vectors $x_1, x_2, ..., x_n$ with associated covariance matrices $C_1, C_2, ..., C_n$, the IF calculates

$$\begin{aligned} x_w &= C_w \sum C_i^{-1} x_i \\ C_w^{-1} &= \sum C_i^{-1} \end{aligned}$$

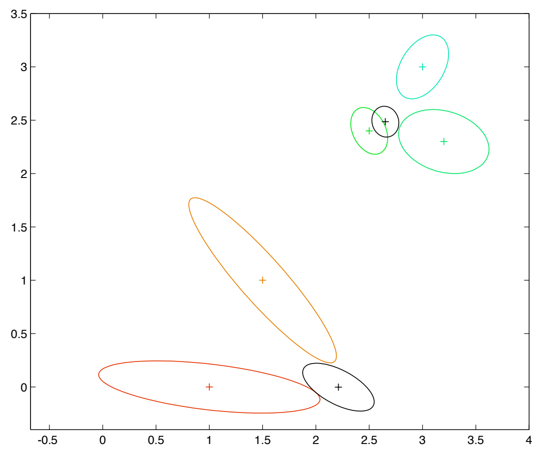

Figure 12: `meanwm.m`

Example: The Matlab code

```
x1 = [1.5; 1]; C1 = [0.08 -0.082; -0.082 0.1];
drawprobellipse(x1,C1,0.95,[.9 .5 0]);
plot(x1(1),x1(2),'+','Color',[.9 .5 0]);
x2 = [1; 0]; C2 = [0.18 -0.02; -0.02 0.01];
drawprobellipse(x2,C2,0.95,[.9 .2 0]);
plot(x2(1),x2(2),'+','Color',[.9 .2 0]);

xin = cat(2,x1,x2); Cin = cat(3,C1,C2);
[xw,Cw] = meanwm(xin,Cin);
drawprobellipse(xw,Cw,0.95,'k');
plot(xw(1),xw(2),'k+');

x3 = [3; 3]; C3 = [0.01 0.005; 0.005 0.015];
drawprobellipse(x3,C3,0.95,[0 .9 .7]);
plot(x3(1),x3(2),'+','Color',[0 .9 .7]);
```

```
x4 = [3.2; 2.3]; C4 = [0.03 -0.005; -0.005 0.015];
drawprobellipse(x4,C4,0.95,[0 .9 .4]);
plot(x4(1),x4(2),'+','Color',[0 .9 .4]);
x5 = [2.5; 2.4]; C5 = [0.005 -0.002; -0.002 0.008];
drawprobellipse(x5,C5,0.95,[0 .9 .1]);
plot(x5(1),x5(2),'+','Color',[0 .9 .1]);

xin = cat(2,x3,x4,x5); Cin = cat(3,C3,C4,C5);
[xw,Cw] = meanwm(xin,Cin);
drawprobellipse(xw,Cw,0.95,'k');
plot(xw(1),xw(2),'k+');
```

generates the figure shown below. Note how the command `cat` is used to prepare the input arguments.

See also `mean`, `cat`, `drawprobellipse`.